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Developing a Non-linear Model for Water and Waste Load Allocation in the River Systems Using Fuzzy Cooperative Game: A Case Study

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Keywords

Water and waste load allocation,
Fuzzy cooperative game,
Cooperative coalition,
Dez river system.

Abstract

In this paper, a model is presented for water and waste load allocation in the river systems under the uncertainty and based upon three models as initial allocation, cooperative allocation and fuzzy cooperative game. The presented method consists of three main stages. First, the initial allocation model of the water and waste load is formulated. Then, the cooperative allocation model of the water and waste load is compiled by organizing all the possible coalitions in order to increase the economic interests. Finally, in order to allocate the water and waste load in the case of cooperation among the water users and with the consideration of uncertainties, the benefits of coalition are reallocated by implementing a fuzzy cooperative game. The capability of the suggestive methodology is verified using the quantitative and qualitative data of Dez river located in Khuzestan, Iran. The obtained results indicate the proper performance of the present model in cooperative allocating of the water and waste loads, maintaining the river's water quality standards and the effect of the formation of the cooperative farming coalitions on their ultimate benefit increase.

1. Introduction

The problem of concurrent allocation of the water and waste loads in the rivers is of high importance in the planning and sustainable management of water resources. With this insight, one can consider the effect of water allocation on the waste loads and also the capacity of river pollution acceptance while allocating the water with the aim of production increase. Many researchers considered the qualitative modeling of water as well as its quantitative one as essential subjects in water allocation.

The number of researches dealing with the simultaneous allocation of water and waste loads is limited. Azevedo et al. [1] implemented a linked of QUAL2E-UNCAS (quality model) and MODSIM (quantity model) for the water qualitative and quantitative modeling in Piracicaba river basin located in Brazil. Also, the uncertainty analysis of model parameters using reliability, vulnerability and resiliency criteria has been performed in this research. The results of this study showed that among the six management options, one can overcome the inappropriate quality of water by increasing the wastewater treatment and also water flow in the river. The weak part of the above study is the lack of attention to the waste load allocation problem. A deterministic model for the quantitative and qualitative water allocations has been presented by Zhang et al. [2]. They implemented a water quality simulation model and 1-D hydrodynamic model in order to consider the pollutants transport, water needs, water supply and hydrological cycling processes in the Jiaojiang watershed basin. In their model, to solve the optimization problem of water allocation, the river is divided into a set of intervals and the surrounding near each interval is considered as a tank. Also, the exchanges between the tanks and rivers are taken into consideration. The uncertainties are not included in this research.

Nikoo et al. [3] used a non-linear interval programming (NIP) approach for solving the problem of uncertain water and waste load allocation and to achieve the water quality standard in the river systems. Further to the development of the initial allocation model of water harvesting license and waste discharge in Dez river, they used its results to develop several models of cooperative games and reallocation for the total profit of the coalitions. In their model, the waste flow is considered as a constant coefficient of the allocated water. Tavakoli et al. [4] presented an uncertain model for the water and waste load allocation in the river system with the simultaneous modelling of the quality and quantity of

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TB	maximum total benefit of the system assuming that the water need of users is completely provided	GWL	initial groundwater level (m)
$\bar{d}_1$ / $\bar{d}_2$	points of disagreement in the Nash function whose values considered as zero in numerous researches	k	hydraulic conductivity coefficient (m/d)
A_i	acreage of the *i*th agricultural water user	Cdr_i	concentration of the water quality index in the *i*th agricultural user's return flow calculated by SWAP model (mg/L)
c_{di}	Cost per deviation of the *i*th user's polluted return flow to the evaporation pond during the planning period	GWQ	concentration of the water quality index in the groundwater (mg/L)
CPD_i	Long-term average annual profit generated by the *i*th water user	c_{up}	concentration of the water quality index in the river's upstream (mg/L)
k_{yi}	product response factor for the *i*th agricultural water user	x_i	water allocated to the *i*th agricultural user (MCM/day)
d_i	daily water need of the *i*th user (MCM/day)	x_{di}	waste load deviated to the evaporation pond by the *i*th agricultural user (MCM/day)
q_{up}	daily flow of the river's upstream (m^3/s)	U_d	water demand of urban area (MCM/day)
ω_u	return flow percent associated to the *i*th water user	E_d	environmental water requirement (MCM/day)
c_k	density of the water quality index in the *k*th check point (mg/L)	f	estimated function by SWAP model indicating the return flow quantity
c_s	standard density of water quality index (mg/L)	g	estimated function by SWAP model indicating the return flow quality

the agricultural return flows. In this model, using the SWAP simulation model, the quality and quantity of the agricultural return flows are considered. Also, in order to control the waste loads, an amount of the return flow of each agricultural is deviated to the evaporation pond. The objective function of this model is the allocation of water to each water user proportional to their water need. In the abovementioned research, no attention is paid to the reallocation of the profit among the water users and cooperation among them.

In addition to the features of the recent publications, in the present water and waste load allocation model, the fairness and economy criteria are concurrently taken into consideration using Nash function. Furthermore, the cooperative allocation model of the water and waste load is compiled by organizing all the possible coalitions in order to increase the economic interests. In the following and with the aid of a fuzzy cooperative game, the benefits of coalition are reallocated with the consideration of the uncertainties.

2. The Proposed Model's Structure

Figure 1 presents the structure of the proposed model for developing the concurrent allocation of water and waste load in the river system using the cooperative fuzzy games theory. The suggested method in this article is including four main steps of data collection, initial allocation model development, possible coalitions' formation and water and waste load cooperative model development by implementing the linear optimization model and finally benefit reallocation using the fuzzy cooperative game model.

In the first step, the required data such as the quality and quantity of water in the river's upstream, water needs of users, quality and quantity of waste loads of various water users, drainage System features, primary level and groundwater quality are collected. In the next step, the initial water and waste load allocation model is developed considering equity and economy criteria. In order to include the contrast between the two aforementioned criteria, Nash function is applied. Therefore, the cost function of the initial allocation model is nonlinear and the genetic algorithm (GA) optimization method is used for solving the problem. In the following and after the formation of possible coalitions, the cooperative allocation model of water and waste load is developed for each coalition. It is saying that the iterative linear programming (ILP) approach is implemented for solving the nonlinear model of cooperative water and waste load allocation [4]. Finally, a fuzzy cooperative game model is developed and performed in order to reallocate the benefits of coalition among the water users under the uncertainty condition.

Given that the benefit results determined by the initial and cooperative allocation models are used as the input of the fuzzy cooperative games model, the benefits obtained from the initial and cooperative allocation models must be converted to the fuzzy numbers in order to consider the possible uncertainty interval. To this aim, a constant bandwidth (25%) is considered for both sides of the fuzzy numbers.

3. Simulation Model of the Agricultural Return Water

Due to the successful applications of SWAP simulation model during the recent years, this model is used in the present research for determining the quantity and quality of the agricultural return flow. The SWAP simulation model simulates the vertical movement of the water and salt in the soil together with the plant growth. The unstable and unsteady flow of water in the soil is evaluated based upon

Richards equation considering the water absorption by the plant as [5]

$$C_w(h)\frac{\partial h}{\partial t} = \frac{\partial}{\partial t}\left[k(h)(\frac{\partial h}{\partial z}+1)\right] - S(z) \qquad (1)$$

where C_W denotes the soil's water capacity (cm^{-1}), h , the soil suction (cm), k , hydraulic conductivity (m/d), S , water absorption parameter ($cm^{-3}cm^{-3}d^{-1}$), t , time (d) and Z is the depth (m). In the SWAP simulation model, the numerical solution of the above equation in achieved based on the soil humidity initial conditions, boundary conditions on the soil's surface and below and the governing relations among the hydraulic parameters of soil including humidity, suction and hydraulic conductivity.

In this research, the calibrated SWAP models are substituted by meta-model to decrease the runtime of the suggested simulation-optimization model. In this step, the SWAP model associated with each agricultural area is frequently executed for various amounts of the allocated water and two polynomial functions are fitted to the simulation results. The meta-model of each agricultural area indicates the relation between the quantity and quality of the return flow and the allocated water. More details on the SWAP simulation model and how to build the meta-models can be found in Tavakoli et al. [4].

4. Optimization Model of the Initial Water and Waste Load Allocation

In this optimization model, the amount of water allocated to each water user and the return flow deviated to the evaporation ponds are assumed as the design variables. In order to control the waste loads and provide the required water quality in the quality control points along the river, an amount of the return flow of each agricultural user is deviated to the evaporation pond. In the present research, the cost function of the optimization model is based on equity and economy criteria. While formulating the cost function, Nash function is implemented to consider the contrast between these two criteria. A schematic view of a river is plotted in Figure 2 which can be used for determining the following optimization variables

$$Max\,Z = (f_1 - \bar{d}_1)(f_2 - \bar{d}_2) \qquad (2)$$

under the following constraints

$$f_1 = \sum_{i=1}^{2}\left\{\left[CPD_i \times d_i \times \left(1-(1-k_{yi}\times(1-\frac{x_i}{d_i}))\right)\times A_i)\right] - (c_{di}\times x_{di})\right\}\times\frac{1}{TB} \qquad (3)$$

$$f_2 = \sum_{i=1}^{2}\left\{\left[CPD_i \times d_i \times \left(1-(1-k_{yi}\times(1-\frac{x_i}{d_i}))\right)\times A_i)\right] - (c_{di}\times x_{di})\right\}\times\frac{1}{d_i} \qquad (4)$$

$$Dr_i = f(x_i, GWL, k) \quad \forall i = 1,2 \qquad (5)$$

$$Cdr_i = l(x_i, GWL, k, GWQ, c_{up}) \quad \forall i = 1,2 \qquad (6)$$

$$\sum_{i=1}^{2}(x_i - Dr_i) + \sum_{i=1}^{2}x_{di} \le q_{up} - (1-\omega_u)U_d - E_d \qquad (7)$$

$$x_1 \le q_{up} - (1-\omega_u)U_d \qquad (8)$$

$$x_2 \le q_{up} - (1-\omega_u)U_d - (x_1 - Dr_1) - x_{d1} \qquad (9)$$

$$0 \le x_i \le d_i \quad \forall i = 1,2 \qquad (10)$$

$$0 \le x_{di} \le Dr_i \quad \forall i = 1,2 \qquad (11)$$

$$c_s \le c_k \quad \forall k = 1,2 \qquad (12)$$

In the above equations, the variables and parameters are defined as in the nomenclature. As mentioned before, the cost function in this optimization model is a kind of Nash function which is a non-linear one. Also, to use the Nash function in the economy criterion estimation, the profit obtained from the agricultural production [6] is divided by the total system profit. Further to these, unlike the previous studies, the equity criterion in the present optimization model is considered to be the profit over demand fraction (the equity allocation in this work is considered to make the maximum benefit of different water user's closer, proportional to their water demands). This is due to the consideration of the deviated waste load in the cost function. GA is implemented for the present non-linear optimization problem.

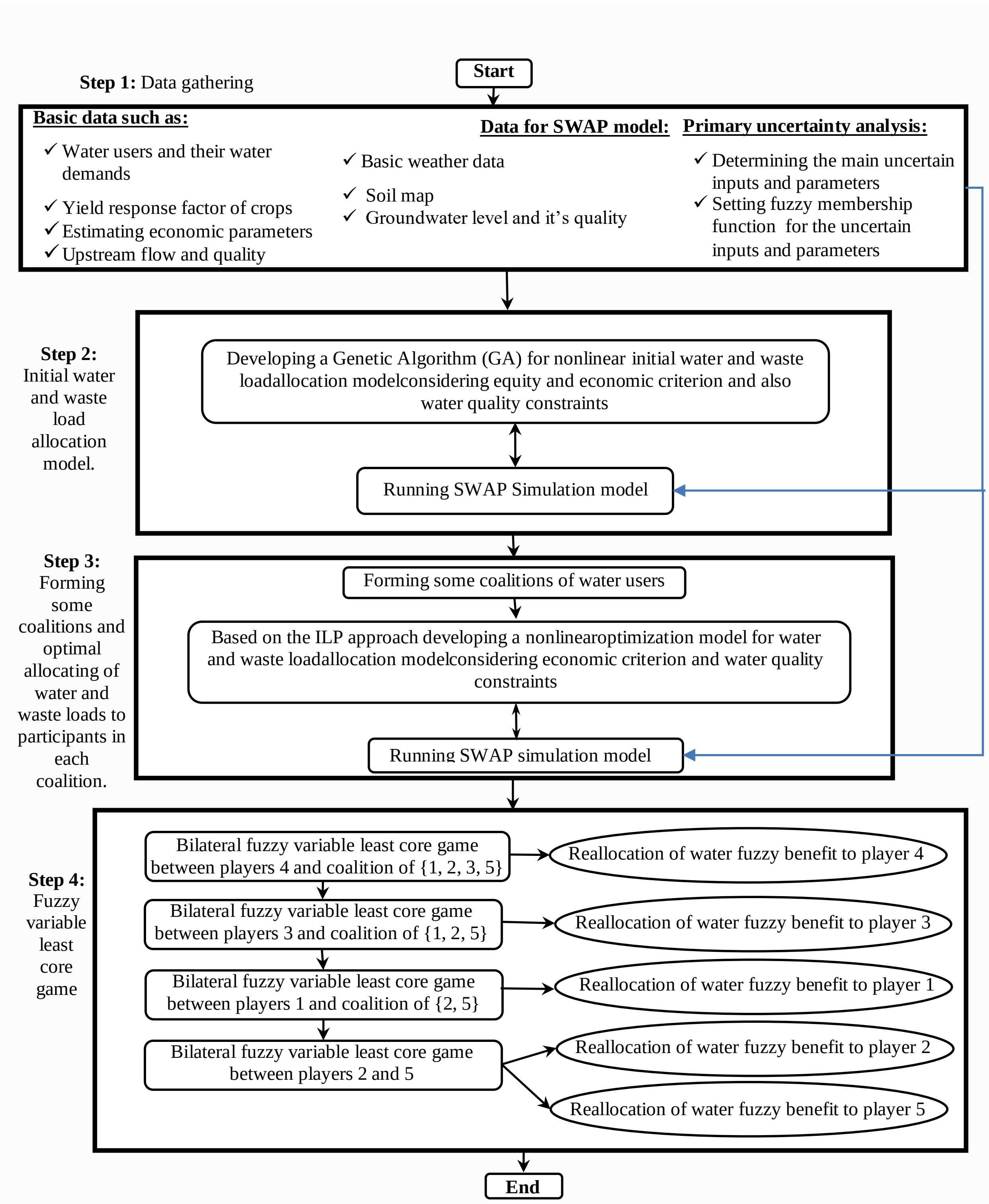

Figure 1. The proposed model structure for qualitative and quantitative water allocation based on fuzzy cooperative game theory

5. The Nonlinear Optimization Model of the Cooperative Water and Waste Load Allocation

In this step, first, all the possible coalitions among the agricultural users are formed and then using a nonlinear optimization model, the water and pollution discharge permit are allocated to those agricultural users participated in the coalition. The amount of allocated water to the agricultural users and deviated return flow to the evaporation ponds are determined with the aim of the total coalition profit maximization. The objective function is economically and the agricultural production function is used in this regard.

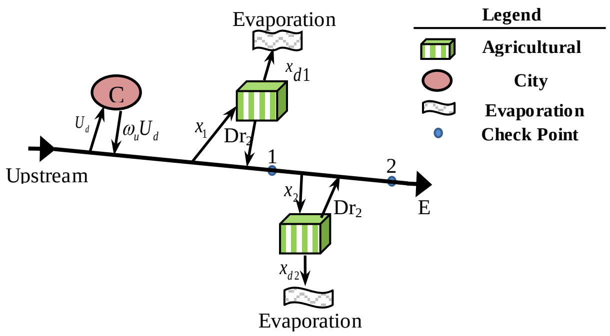

Figure 2. Schematic view of the river system usable for the optimization model

Considering the participation of the agricultural users in the S coalition and due to Figure 2, the optimization model can be formulated as below

$$Max\,Z = \sum_{i=1}^{2} B_i'^{net} = \sum_{i=1}^{2} \left\{ \left[CPD_i \times d_i \times \left(1 - (1 - k_{yi} \times (1 - \frac{x_i}{d_i})) \right) \times A_i) \right] - (c_{di} \times x_{di}) \right\} \tag{13}$$

with the following constraints

$$\sum_{i \in S} B_i'^{net} \geq \sum_{i \in S} B_i^{net} \qquad \forall i \in S \tag{14}$$

$$B_i'^{net} = B_i^{net} \qquad \forall i \notin S \tag{15}$$

in which S stands for a coalition of water users, B_i^{net} is the profit corresponding to the ith profit obtained via initial allocation model and $B_i'^{net}$ defines the same value achieved from cooperative allocation model. The profit of the user that does not participate in the coalition must be equal to the profit obtained from the initial allocation model (Eq. (15)). On the other hand, the sum of the profits corresponding to the other users which are present in the coalition must be equal or more than the profit obtained via the initial allocation model (Eq. (14)). The other constraints for the cooperative allocation optimization model are the same as those of the initial allocation one (Eqs. (3)-(14)). It is saying that in the present optimization model, the water quality in the qualitative control point 2 depends on the water quality at control point 1 and the water amount allocated to the user 2. Therefore, some of the constraints relative to the water quality are nonlinear. The ILP method is applied for solving the resultant nonlinear model [4].

6. Profit Allocation Using the Nucleus Game with Fuzzy Variables

In the present study, the concept of Nucleus cooperative games theory is used in order to apply justice between stakeholders. Jafarzadegan et al. suggested a new model of fuzzy variable least core game. In their model, first, fuzzy relations are developed for the Nucleus two-player game. Then, a new algorithm presented by them upon using, a multi-player game is converted to several two-player games. In this way, the fuzzy cooperative game can be extended for any number of players [7]. In this model, the stakeholders are entered to the game by fuzzy profitability and finally the allocated profits to them are also obtained in terms of fuzzy variables. Also in this study, the Nucleus game approach with fuzzy variables is implemented for reallocating the profit obtained via the initial and cooperative allocation models. For further details on the Nucleus game approach with fuzzy variables, the reader is referred to Jafarzadegan et al. [7].

7. Case Study

In order to verify the capability of the proposed methodology in water and waste allocation problem of river systems, Dez river system is chosen as the case study. This river system belongs to Karun river basin. The area under study includes Dez river from Dez dam downstream, Dezful city and five agricultural water users. The schematic view of the system in the under study area is illustrated by Figure 3. The annual water requirement average of Dezful city is about 65.9 millions cubic meters (MCM). Also, the environmental monthly water requirement of Dez river is about 240 MCM.

In this research, water demand of Dezful city is completely provided regarding to the strategic condition of water demand provision and environmental needs. In the above study area, sugarcane is the dominant culture. Total dissolved solids (TDS) is considered as the river water quality index in the present study. Figure 4 depicts the water demand of the water users. Also, Table 1 lists the data of average discharge flow and TDS concentration in the return flow of the five agricultural users.

Table 1. Discharge flow and TDS concentration associated with five agricultural water users

Water user	TDS concentration (mg/l)	Discharge flow (m3/s)
1	1220	5.54
2	4900	2.53
3	1860	9.516
4	2220	9.313
5	5770	0.893

8. Results and Discussions

In order to save runtime in running the developed simulation-optimization model, the calibrated SWAP models are replaced by five meta-models. To this aim, a polynomial function is fitted to the results obtained via system simulation using the SWAP model. In this step, the SWAP model corresponding to each agricultural area is executed repeatedly for different values of the allocated water and two polynomials are fitted to the simulated findings. For example, the results achieved via the SWAP simulation model for the water user number 5 and its corresponding non-linear meta-model are plotted in Figure 5.

Due to the irrigation period of sugarcane in the study area, the optimization model is executed for ten days (in May when the river water has the worst quality). The amounts of the allocated water to each water user and the deviated waste load to the evaporation pond which have been found from the initial and cooperative allocation models are presented in Table 2. According to these water and waste load allocations, TDS density is always lower than 1000 mg/L.

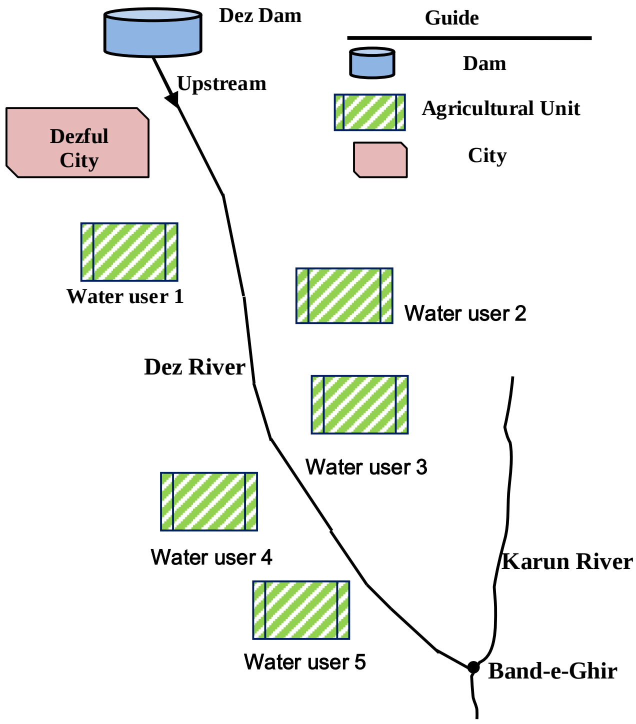

Figure 3. Schematic view of Dez river, main water users, Dezful city and study area

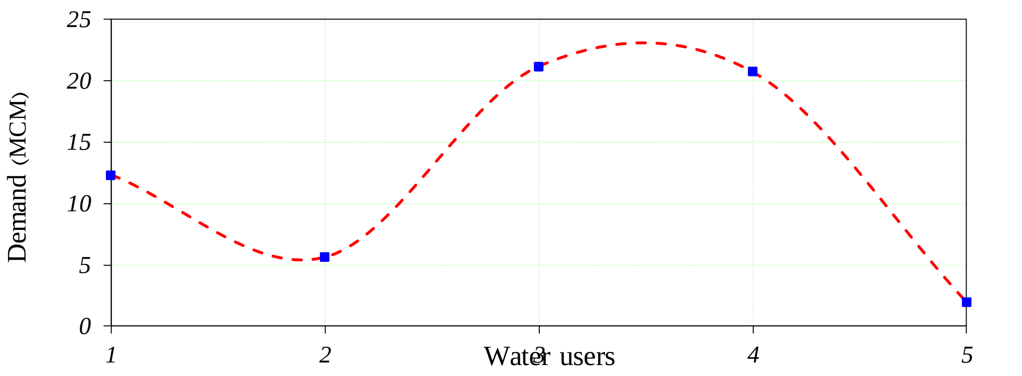

Figure 4. Average water demand of agricultural water users in the study area in May (MCM)

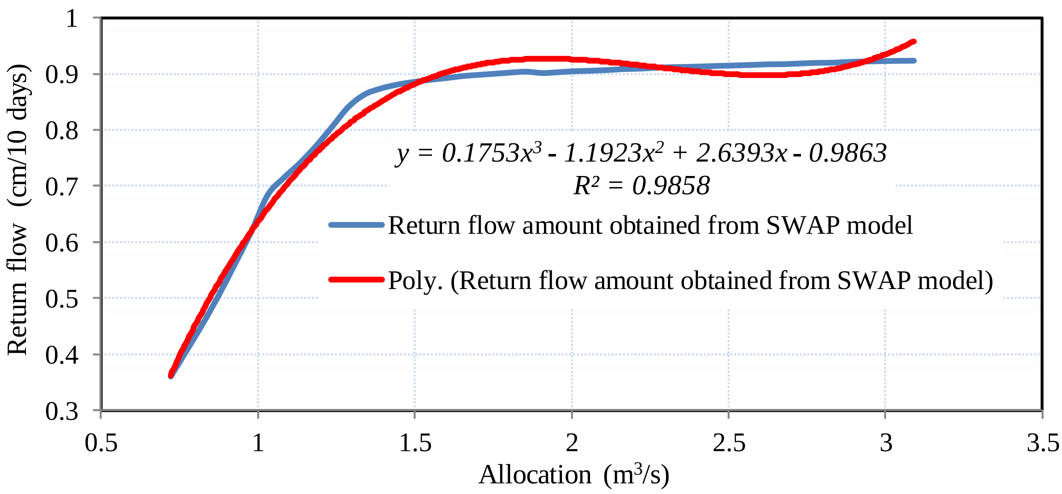

Figure 5. The amount of return flow obtained via the SWAP and fitted regression models

Table 2. The amount of users' allocated water and the corresponding deviated waste loads to the evaporation pond during the planning period (MCM)

Water user	Cooperative allocation model (big coalition)		Initial allocation model	
	Waste load (MCM)	Allocation (MCM)	Waste load (MCM)	Allocation (MCM)
1	0.00032	11.37	0.11	11.37
2	0.00001	2.95	0.36	1.86
3	0.00011	19.54	0.66	9.86
4	0.00024	19.12	0.18	19.12
5	0.000012	0.99	0.14	0.84

To determine the allocated fuzzy profits to the five agricultural water users, first, the fuzzy profitability of water users providing their independent activities are calculated with respect to the initial allocation model. Then using the cooperative allocated model, the fuzzy profitability of all possible coalitions is estimated. These estimated fuzzy values are given in Table 3.

Table 3. Profits of the agricultural water users and coalitions (10^5\$)

Big coalition	Coalitions' profitability			Water users' profitability (players)				
{1,2,3,4,5}	{1,2,3,5}	{1,2,5}	{2,5}	4	3	1	2	5
(120, 46)	(69, 27.5)	(21, 8)	(3.5, 1.5)	(49, 19)	(46, 18)	(17, 6)	(3, 1.5)	(0.5, 0.5)

All the fuzzy values of Table 3, form the inputs for the Nucleus cooperative game with fuzzy variables. Finally, the fuzzy allocated profit to each of the five water users (players) during the simulation-optimization period are obtained as the final outputs of the fuzzy cooperative game. Table 4 lists the ultimate fuzzy allocated profits to the different agricultural water users (players). In this table, the left hand side values indicate the profit while the right hand side ones define the fuzziness of the profits (for example 50 and 19.75 stand for the profit and its fuzziness of the water user number 4, respectively).

Table 4. Ultimate fuzzy allocated profits to the agricultural users in the Dez river system during the short term period (10^5 \$)

water user 1	water user 2	water user 3	water user 4	water user 5
(17.75, 6.1325)	(3.625, 1.8063)	(47.5, 18.625)	(50, 19.75)	(1.125, 0.5063)

Table 4 indicates the remarkable increase in the water users' profit caused by the cooperation. Also, the degree of fuzziness of the results has been increased proportional to the profit increase. It can be concluded that the results obtained illustrate the capability of the suggested methodology and the formation of cooperative coalitions among agricultural users can significantly affect their ultimate profit increase.

9. Conclusions

A new water and waste load allocation model was presented in this paper for water and waste load allocation in the river systems under uncertainty which based upon three models as initial allocation, cooperative allocation and fuzzy cooperative game is presented. In addition to the features of the recent publications, in the present water and waste load allocation model, the equity and economy criteria are taken into account using Nash function. Furthermore, the cooperative allocation model of the water and waste load is compiled by participating all the possible coalitions in order to increase the economic interests. Efficiency and applicability of the methodology was examined using data obtained from the Dez river system in south-west Iran. Results showed the capability of the methodology for water and waste load allocation in rivers under uncertainty. The obtained results indicate the proper performance of the present model in cooperative allocating of the water and waste loads, maintaining the river's water quality standards.

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